Exam 1 Review Solutions

MA 265

February 2021

1. $A = \begin{bmatrix} 3 & 4 & -3 \\ 2 & 5 & 5 \\ -2 & -3 & 1 \end{bmatrix}$

(a) Lots of possibilities for an echelon form. One is $\begin{bmatrix} 3 & 4 & -3 \\ 0 & 7 & 21 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $(1,1)$, $(2,2)$ entries. First and second columns.

(d) Inconsistent.

(e) No.

(f) No.

(g) No. No.

(h) $\left\{ \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix} \right\}$

(i) $\left\{ \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ -3 \end{bmatrix} \right\}$

(j) 1.

(k) 2.

2. $A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$.

(a) Lots of possibilities for an echelon form. One is $\begin{bmatrix} 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \\ 0 & 0 & 0 & 0 & 2 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

(c) $(1,1)$, $(2,2)$, $(3,5)$ entries. First, second, and fifth columns.

(d) Consistent. $x = \begin{bmatrix} 13 \\ 3 \\ 0 \\ 0 \\ -5/2 \\ 0 \end{bmatrix} + r \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 24 \\ 7 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$

(e) No.

(f) Yes.

(g) No. Yes.

(h) $\left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 24 \\ 7 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

(i) $\left\{ \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \\ -9 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 6 \end{bmatrix} \right\}$

(j) 3.

(k) 3.

3. $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 2 & -2 \\ 2 & 1 & -3 \end{bmatrix}$.

(a) Lots of possibilities for an echelon form. One is $\begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & -5 \\ 0 & 0 & -11 \end{bmatrix}$.

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $(1,1)$, $(2,2)$, $(3,3)$ entries. First, second, and third columns.

(d) Consistent. $x = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

(e) Yes.

(f) Yes.

(g) Yes. Yes.

(h) $\{0\}$

(i) $\left\{ \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \right\}$

(j) 0.

(k) 3.

4. 4

5. 4

6. Yes. $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \frac{9}{4} \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} - \frac{7}{4} \begin{bmatrix} 0 \\ 8 \\ 5 \end{bmatrix} + \frac{5}{4} \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

7. Linearly dependent.

$$-\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = 0$$

If linearly dependent, provide an equation of linear dependence.

8. Let $\{v_1, \ldots, v_n\}$ be a set of vectors of $\mathbb{R}^m$.

(a) Yes, but not always.

(b) No.

(c) No.

(d) Yes, but not always.

9. $\frac{7}{20}$

10. $\begin{bmatrix} \frac{3}{20} & \frac{1}{20} & \frac{-3}{10} \\ \frac{3}{40} & \frac{1}{40} & \frac{7}{20} \\ \frac{1}{8} & \frac{-1}{8} & \frac{-1}{4} \end{bmatrix}$

11. 11

12. $\operatorname{rank} A = 4$ and $\dim \operatorname{Nul} A = 2$

13. Yes. $B^{-1} = \frac{1}{-33} \begin{bmatrix} 1 & -7 \\ -5 & 2 \end{bmatrix}$.

14. -32.

15. Let T be the linear transformation with standard matrix $\begin{bmatrix} 1 & -3 & 6 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 7 \end{bmatrix}$.

(a) False

(b) True

(c) True

(d) False

16. $\begin{bmatrix} -3 & 2 & 5 \\ 6 & 12 & 6 \\ 13 & -1 & -14 \\ 23 & 0 & 8 \end{bmatrix}$

17. $\det \begin{bmatrix} 4 & 6 & 3 & 2 \\ 0 & 2 & 8 & 19 \\ 0 & 0 & 1 & -13 \\ 0 & 0 & 0 & -2 \end{bmatrix} = -16 \qquad \text{and} \qquad \det \begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{bmatrix} = 0$

18. Column space has basis $\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$. The null space has basis $\left\{ \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix} \right\}$. A is not invertible.

19. Find the matrices for the following linear transformations:

(a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$